One-Way BG ANOVA

Andrew Ainsworth
Psy 420

Topics

- Analysis with more than 2 levels
  - Deviation, Computation, Regression, Unequal Samples
- Specific Comparisons
  - Trend Analysis, Planned comparisons, Post-Hoc Adjustments
- Effect Size Measures
  - Eta Squared, Omega Squared, Cohen's *d*
- Power and Sample Size Estimates

Deviation Approach

$$SS_T = \sum \left(Y_{ij} - GM\right)^2$$

$$SS_A = n\sum \left(\bar{Y}_j - GM\right)^2$$

$$SS_{S/A} = \sum \left(Y_{ij} - \bar{Y}_j\right)^2$$

- When the n's are not equal

$$SS_A = \sum n_j \left(\bar{Y}_j - GM\right)^2$$

Analysis - Traditional

- The traditional analysis is the same

$$SS_Y = SS_T = \sum Y^2 - \frac{\left(\sum Y\right)^2}{N} = \sum Y^2 - \frac{T^2}{an}$$

$$SS_A = \frac{\sum \left(\sum a_j\right)^2}{n} - \frac{T^2}{an}$$

$$SS_{S/A} = \sum Y^2 - \frac{\sum \left(\sum a_j\right)^2}{n}$$

Analysis - Traditional

- Traditional Analysis – Unequal Samples

$$SS_A = \left(\frac{\left(\sum A_i\right)^2}{n_i} + \frac{\left(\sum A_j\right)^2}{n_j} + \cdots + \frac{\left(\sum A_k\right)^2}{n_k} \right) - \frac{T^2}{N}$$

$$SS_{S/A} = \sum Y^2 - \left(\frac{\left(\sum A_i\right)^2}{n_i} + \frac{\left(\sum A_j\right)^2}{n_j} + \cdots + \frac{\left(\sum A_k\right)^2}{n_k} \right)$$

Unequal N and DFs

$$df_{total} = N - 1 = (n_1 + n_2 + n_3 + \cdots + n_k) - 1$$

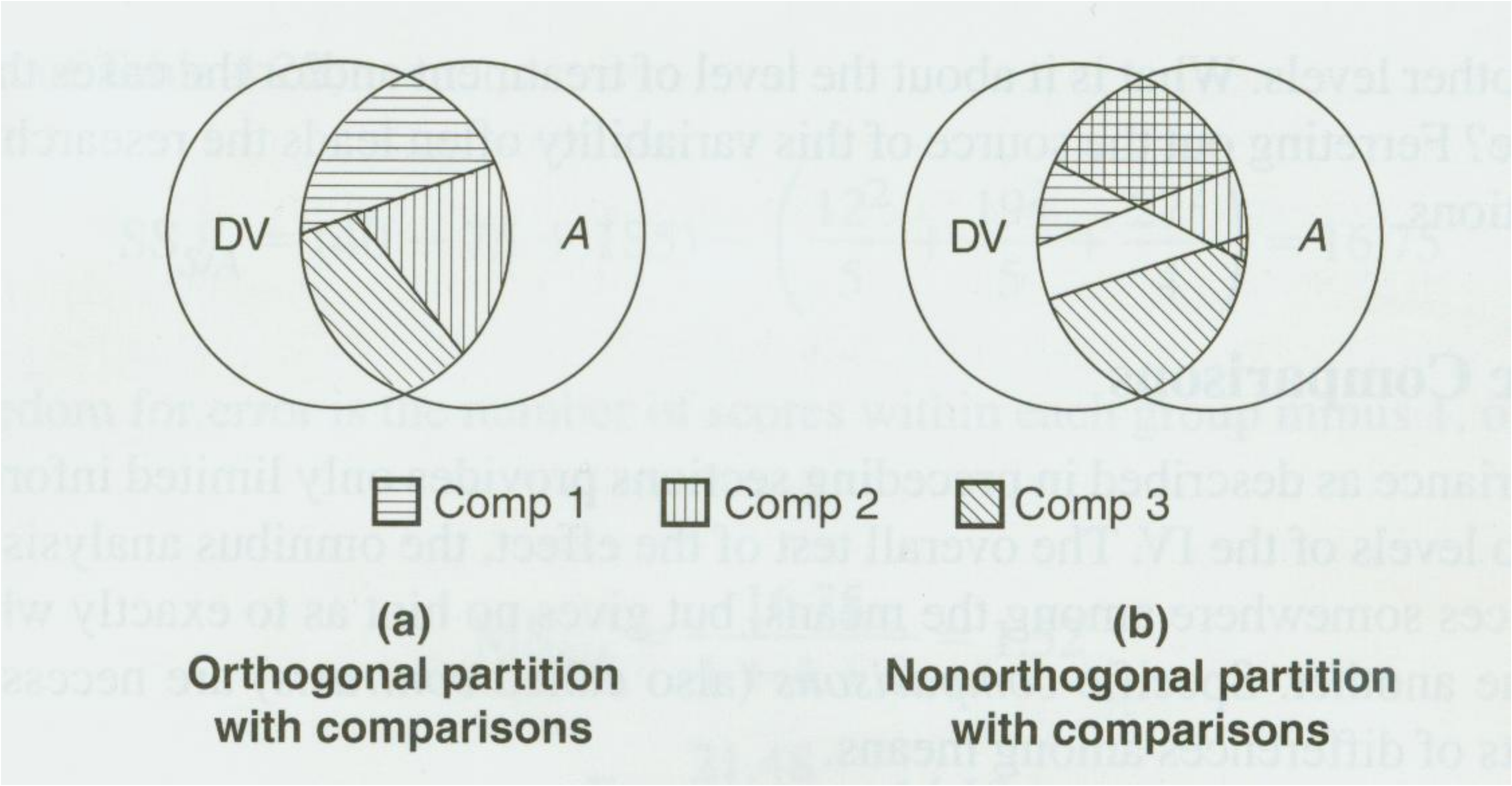
$$df_A = a - 1$$

$$df_{S/A} = (n_1 - 1) + (n_2 - 1) + (n_3 - 1) + \cdots + (n_k - 1)$$

Analysis - Regression

- In order to perform a complete analysis of variance through regression you need to cover all of the between groups variance
- To do this you need to:
  - Create k – 1 dichotomous predictors (Xs)
  - Make sure the predictors don't overlap

Analysis – Regression



Analysis – Regression

- One of the easiest ways to ensure that the comps do not overlap is to make sure they are orthogonal
  - Orthogonal (independence)
    - The sum of each comparison equals zero
    - The sum of each cross-product of predictors equals zero

Analysis – Regression

Level of A	Case	Y	X_1	X_2	Y^2	X_1^2	X_2^2	YX_1	YX_2
Control	s_1	8							
	s_2	7							
	s_3	9							
	s_4	9							
	s_5	7							
Ativan	s_6	8							
	s_7	8							
	s_8	6							
	s_9	7							
	s_{10}	7							
Scruital	s_{11}	4							
	s_{12}	5							
	s_{13}	4							
	s_{14}	7							
	s_{15}	4							
	Sum	100							
	N	15							
	Mean	6.67							

Analysis - Regression

Level of A	Case	Y	X_1	X_2	Y^2	X_1^2	X_2^2	YX_1	YX_2
Control	s_1	8	2						
	s_2	7	2						
	s_3	9	2						
	s_4	9	2						
	s_5	7	2						
Ativan	s_6	8	-1						
	s_7	8	-1						
	s_8	6	-1						
	s_9	7	-1						
	s_{10}	7	-1						
Scruital	s_{11}	4	-1						
	s_{12}	5	-1						
	s_{13}	4	-1						
	s_{14}	7	-1						
	s_{15}	4	-1						
	Sum	100							
	N	15							
	Mean	6.67							

Analysis - Regression

Level of A	Case	Y	X_1	X_2	Y^2	X_1^2	X_2^2	YX_1	YX_2	X_1X_2
Control	s_1	8	2	0						0
	s_2	7	2	0						0
	s_3	9	2	0						0
	s_4	9	2	0						0
	s_5	7	2	0						0
Ativan	s_6	8	-1	1						-1
	s_7	8	-1	1						-1
	s_8	6	-1	1						-1
	s_9	7	-1	1						-1
	s_{10}	7	-1	1						-1
Scruital	s_{11}	4	-1	-1						1
	s_{12}	5	-1	-1						1
	s_{13}	4	-1	-1						1
	s_{14}	7	-1	-1						1
	s_{15}	4	-1	-1						1
	Sum	100	0	0						0
	N	15								
	Mean	6.67	0	0						

Analysis – Regression

Level of A	Case	Y	X_1	X_2	Y^2	X_1^2	X_2^2	YX_1	YX_2	X_1X_2
Control	s_1	8	2	0	64	4	0	16	0	0
	s_2	7	2	0	49	4	0	14	0	0
	s_3	9	2	0	81	4	0	18	0	0
	s_4	9	2	0	81	4	0	18	0	0
	s_5	7	2	0	49	4	0	14	0	0
Ativan	s_6	8	-1	1	64	1	1	-8	8	-1
	s_7	8	-1	1	64	1	1	-8	8	-1
	s_8	6	-1	1	36	1	1	-6	6	-1
	s_9	7	-1	1	49	1	1	-7	7	-1
	s_{10}	7	-1	1	49	1	1	-7	7	-1
Scruital	s_{11}	4	-1	-1	16	1	1	-4	-4	1
	s_{12}	5	-1	-1	25	1	1	-5	-5	1
	s_{13}	4	-1	-1	16	1	1	-4	-4	1
	s_{14}	7	-1	-1	49	1	1	-7	-7	1
	s_{15}	4	-1	-1	16	1	1	-4	-4	1
	Sum	100	0	0	708	30	10	20	12	0
	N	15								
	Mean	6.67	0	0						

Analysis – Regression

- Formulas

$$SS(Y) = \sum Y^2 - \frac{{\sum Y}^{\;2}}{N}$$

$$SS(X_i) = \sum X_i^2 - \frac{{\sum X_i}^{\;2}}{N}$$

$$SP(YX_i) = \sum YX_i - \frac{\sum Y \quad \sum X_i}{N}$$

$$SP(X_i X_j) = \sum X_i X_j - \frac{\sum X_i \quad \sum X_j}{N}$$

Analysis – Regression

- Formulas

$$SS_{(Total)} = SS_{(Y)}$$

$$SS_{(regression)} = SS_{(reg.X_i)} + SS_{(reg.X_j)} \cdots = \frac{SP(YX_i)^2}{SS(X_i)} + \frac{SP(YX_j)^2}{SS(X_j)} \cdots$$

$$SS_{(residual)} = SS_{(Total)} - SS_{(regression)}$$

$$b_i = \frac{[SP(YX_i)][SS(X_j)]\cdots[SS(X_k)] - [SP(YX_j)]\cdots[SP(YX_k)]}{[SS(X_i)][SS(X_j)]\cdots[SS(X_k)] - [SP(X_iX_j)]^2[SP(X_iX_k)]^2\cdots[SP(X_jX_k)]^2}$$

Analysis – Regression

- Example

$$SS(Y) = 708 - \frac{(100)^2}{15} = 708 - \frac{10,000}{15} = 708 - 666.67 = 41.33$$

$$SS(X_1) = 30 - \frac{(0)^2}{15} = 30$$

$$SS(X_2) = 10 - \frac{(0)^2}{15} = 10$$

Analysis - Regression

- Example

$$SP(YX_1) = 20 - \frac{(100)(0)}{15} = 20$$

$$SP(YX_2) = 12 - \frac{(100)(0)}{15} = 12$$

$$SP(X_1X_2) = 0 - \frac{(0)(0)}{15} = 0$$

Analysis - Regression

- Example

$$SS_{(Total)} = 41.33$$

$$SS_{(reg)} = \frac{20^2}{30} + \frac{12^2}{10} = \frac{400}{30} + \frac{144}{10} = 13.33 + 14.4 = 27.73$$

$$SS_{(res)} = 41.33 - 27.73 = 13.6$$

Analysis - Regression

- Example

$$df_{(Total)} = N - 1 = 15 - 1 = 14$$

$$df_{(reg)} = \#\ predictors = 2$$

$$df_{(res)} = N - a = 15 - 3 = 12$$

Analysis - Regression

- Example

Source	SS	df	MS	F
Reg	27.73	2	13.867	12.235
Res	13.60	12	1.133	
Total	41.33	14		

- $F_{crit}(2,12) = 3.88$, since 12.253 is greater than 3.88 you reject the null hypothesis.
- There is evidence that drug type can predict level of anxiety

Analysis - Regression

- Example

$$b_1 = \frac{20(10) - 12(0)}{30(10) - (0)^2} = \frac{200 - 0}{300 - 0} = .67$$

$$b_2 = \frac{12(30) - 20(0)}{30(10) - (0)^2} = \frac{360 - 0}{300 - 0} = 1.2$$

$$a = \bar{Y} - b_1(\bar{X}_1) - b_2(\bar{X}_2) = 6.67 - .67(0) - 1.2(0) = 6.67$$

$$Y' = 6.67 + .67(X_1) + 1.2(X_2)$$

Analysis - Regression

- SPSS

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.819[a]	.671	.616	1.06458

a. Predictors: (Constant), X2, X1

Analysis - Regression

- SPSS

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	27.733	2	13.867	12.235	.001[a]
	Residual	13.600	12	1.133		
	Total	41.333	14			

a. Predictors: (Constant), X2, X1

b. Dependent Variable: Y

Analysis - Regression

- SPSS

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	6.667	.275		24.254	.000
	X1	.667	.194	.568	3.430	.005
	X2	1.200	.337	.590	3.565	.004

a. Dependent Variable: Y

Specific Comparisons

- F-test for Comparisons

$$F = \frac{n(\sum w_j \bar{Y}_j)^2 / \sum w_j^2}{MS_{S/A}} = \frac{SS_{(reg.X_j)}}{MS_{(resid)}}$$

  - n = number of subjects in each group
  - $\sum w_j \bar{Y}_j{}^2$ = squared sum of the weighted means
  - $\sum w_j$ = sum of the squared coefficients
  - $MS_{S/A}$= mean square error from overall ANOVA

Specific Comparisons

- If each group has a different sample size…

$$F = \frac{(\sum w_j \bar{Y}_j)^2 / \sum (w_j^2 / n_j)}{MS_{S/A}} =$$

Specific Comparisons

- Example

$$F_{(X_1)} = \frac{5[(2)(8)+(-1)(7.2)+(-1)(4.8)]^2 \Big/ [2^2+(-1)^2+(-1)^2]}{1.13}$$

$$= \frac{5[16-7.2-4.8]^2/6}{1.13} = \frac{13.33}{1.13} = 11.8$$

$$F_{(X_2)} = \frac{5[(0)(8)+(1)(7.2)+(-1)(4.8)]^2 \Big/ [0^2+(1)^2+(-1)^2]}{1.13}$$

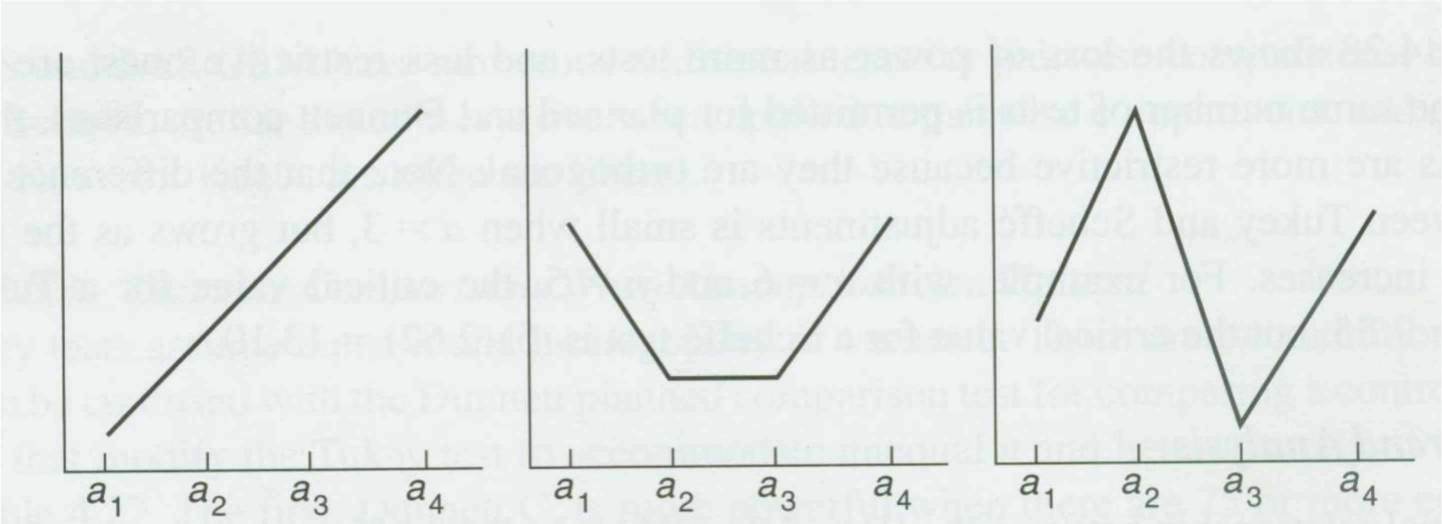
$$= \frac{5[0+7.2-4.8]^2/2}{1.13} = \frac{14.4}{1.13} = 12.74$$

Specific Comparisons

- Trend Analysis
  - If you have ordered groups (e.g. they differ in amount of Milligrams given; 5, 10, 15, 20)
  - You often will want to know whether there is a consistent trend across the ordered groups (e.g. linear trend)
  - Trend analysis comes in handy too because there are orthogonal weights already worked out depending on the number of groups (pg. 703)

Specific Comparisons

- Different types of trend and coefficients for 4 groups



(a) Linear trend (−3, −1, 1, 3)   **(b) Quadratic trend** (1, −1, −1, 1)   **(c) Cubic trend** (−1, 3, −3, 1)

Specific Comparisons

- Mixtures of Linear and Quadratic Trend

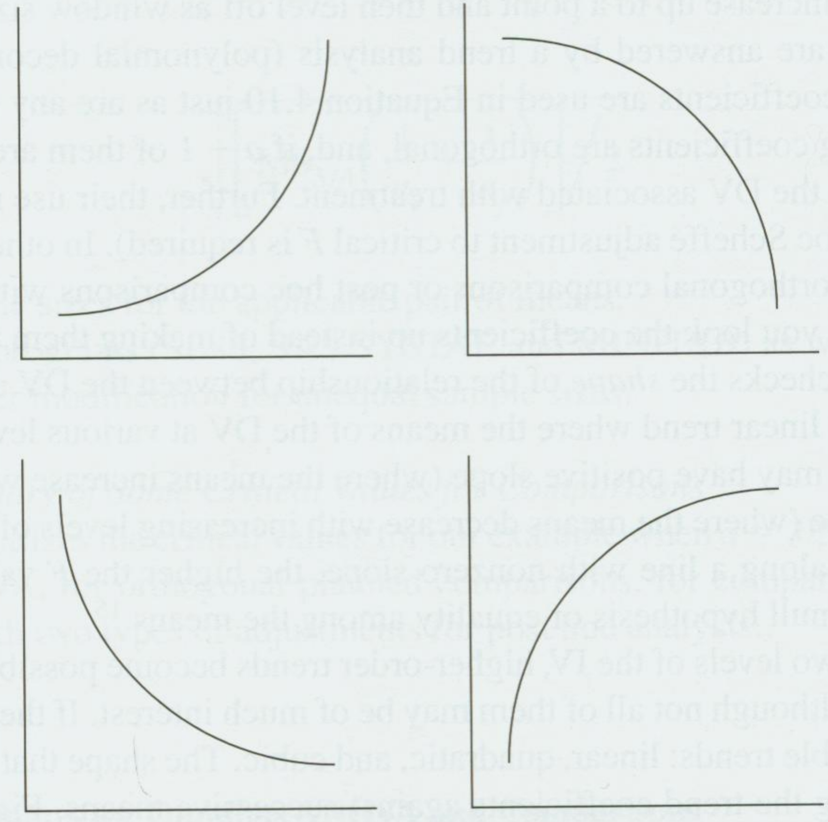


FIGURE 4.4 Shapes Represented by a Combination of Linear and Quadratic Trends

Specific Comparisons

- Planned comparisons - if the comparisons are planned than you test them without any correction
- Each F-test for the comparison is treated like any other F-test
- You look up an F-critical value in a table with df_{comp} and df_{error}.

Specific Comparisons

- Example – if the comparisons are planned than you test them without any correction…

$$F_{crit}(1,12) = 4.75$$

  - F_{x1}, since 11.8 is larger than 4.75 there is evidence that the subjects in the control group had higher anxiety than the treatment groups
  - F_{x2}, since 12.75 is larger than 4.75 there is evidence that subjects in the Scruital group reporter lower anxiety than the Ativan group

Specific Comparisons

- Post hoc adjustments
  - Scheffé
    - This is used for complex comparisons, and is conservative
    - Calculate F_{comp} as usual
    - $F_S = (a – 1)F_C$
      - where F_S is the new critical value
      - a – 1 is the number of groups minus 1
      - F_C is the original critical value

Specific Comparisons

- Post hoc adjustments
  - Scheffé – Example
    - $F_{X1} = 11.8$
    - $F_S = (3 - 1) * 4.75 = 9.5$
    - Even with a post hoc adjustment the difference between the control group and the two treatment groups is still significant

Specific Comparisons

- Post hoc adjustments
  - Tukey's Honestly Significant Difference (HSD) or Studentized Range Statistic
    - For all pairwise tests, no pooled or averaged means
    - F_{comp} is the same
    - $F_T = \frac{q_T^2}{2}$, q_T is a tabled value on pgs. 699-700

Specific Comparisons

- Post hoc adjustments
  - Tukey's Honestly Significant Difference (HSD) or Studentized Range Statistic
    - Or if you have many pairs to test you can calculate a significant mean difference based on the HSD
    - $\bar{d}_T = q_T \sqrt{\frac{MS_{S/A}}{n}}$, where q_T is the same as before
    - $\bar{d}_T = q_T \sqrt{\left[MS_{S/A} \left(\frac{1}{n_i} + \frac{1}{n_j} \right) \right] / 2}$, when unequal samples

Specific Comparisons

- Post hoc adjustments
  - Tukey's – example

$$F_T = \frac{3.77^2}{2} = 7.11$$

- Since 12.74 is greater than 7.11, the differences between the two treatment groups is still significant after the post hoc adjustment

Specific Comparisons

- Post hoc adjustments
  - Tukey's – example
  - Or you calculate:

$$\bar{d}_T = 3.77\sqrt{\frac{1.13}{5}} = 1.79$$

- This means that any mean difference above 1.79 is significant according to the HSD adjustment
- 7.2 – 4.8 = 2.4, since 2.4 is larger than 1.79…

Effect Size

- A significant effect depends:
  - Size of the mean differences (effect)
  - Size of the error variance
  - Degrees of freedom
- Practical Significance
  - Is the effect useful? Meaningful?
  - Does the effect have any real utility?

Effect Size

- Raw Effect size –
  - Just looking at the raw difference between the groups
  - Can be illustrated as the largest group difference or smallest (depending)
  - Can’t be compared across samples or experiments

Effect Size

- Standardized Effect Size
  - Expresses raw mean differences in standard deviation units
  - Usually referred to as *Cohen's d*

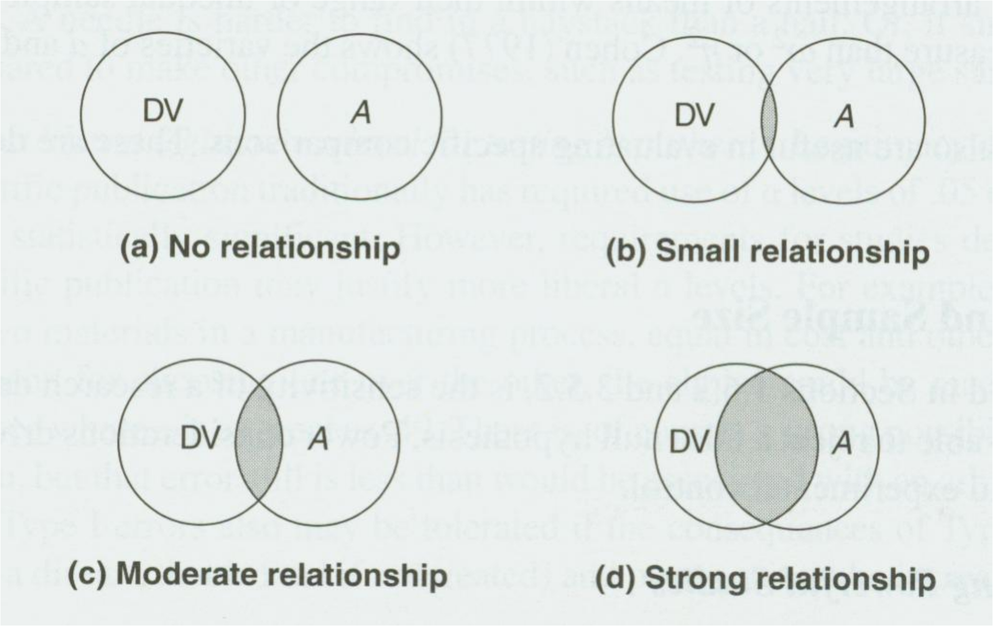
$$d = \frac{\left| \bar{Y}_l - \bar{Y}_s \right|}{\sqrt{MS_{S/A}}}$$

Effect Size

- Standardized Effect Size
  - Cohen established effect size categories
    - .2 = small effect
    - .5 = moderate effect
    - .8 = large effect

Effect Size

- Percent of Overlap
  - There are many effect size measures that indicate the amount of total variance that is accounted for by the effect



Effect Size

- Percent of Overlap
  - *Eta Squared*

$$\eta^2 = R^2 = \frac{SS_A}{SS_T}$$

    - *simply a descriptive statistic*
    - *Often overestimates the degree of overlap in the population*

Effect Size

- *Omega Squared*

$$\hat{\omega}^2 = \frac{SS_A - df_A(MS_{S/A})}{SS_T + MS_{S/A}}$$

  - *This is a better estimate of the percent of overlap in the population*
  - *Corrects for the size of error and the number of groups*

Effect Size

- Example

$$\eta^2 = \frac{27.73}{41.33} = .67$$

$$\hat{\omega}^2 = \frac{27.73 - 2(1.13)}{41.33 + 1.13} = \frac{27.73 - 2.26}{42.46} = \frac{25.47}{42.46} \approx .60$$

Effect Size

- For comparisons
  - You can think of this in two different ways

$$\eta^2 = \frac{SS_{comp}}{SS_T} \; or \; \frac{SS_{comp}}{SS_A}$$

- SS_{comp} = the numerator of the F_{comp}

Effect Size

- For comparisons - Example

$$\eta^2_{X_1} = \frac{13.33}{41.33} = .32$$

or

$$\eta^2_{X_1} = \frac{13.33}{27.73} = .48$$

Power and Sample Size

- Designing powerful studies
  - Select levels of the IV that are very different (increase the effect size)
  - Use a more liberal α level
  - Reduce error variability
  - Compute the sample size necessary for adequate power

Power and Sample Size

- Estimating Sample size
  - There are many computer programs that can compute sample size for you (PC-Size, G-power, etc.)
  - You can also calculate it by hand:

$$n = \frac{2\sigma^2}{\delta^2}(z_{1-\alpha} + z_{1-\beta})^2$$

- Where σ^2 = estimated $MS_{S/A}$
- δ = desired difference
- $Z_{\alpha-1}$ = Z value associated with 1 - α
- $z_{\beta-1}$ = Z value associated with 1 - β

Power and Sample Size

- Estimating Sample size – example
  - For overall ANOVA with alpha = .05 and power = .80 (values in table on page 113)
    - Use the largest mean difference

$$n = \frac{2(1.13)^2}{(8-4.8)^2}(1.96+.84)^2 = \frac{2.55}{10.24}(7.84) = 1.95 \approx 2$$

    - Roughly 2 subjects per group
  - For all differences significant

$$n = \frac{2(1.13)^2}{(8-7.2)^2}(1.96+.84)^2 = \frac{2.55}{.64}(7.84) = 31.23 \approx 31$$

    - Roughly 31 subjects per group